however, that some matters of elementary mathematics could not have been omitted without detriment. Thus the discussion, at the beginning of the second part, of the equations of a conic, based on the definition of a conic as the plane section of a right circular cone, must be superfluous for a reader who is capable of following the whole of the first part intelligently. But the fault is doubtless on the right side.

The whole work consists of seven parts. The first deals with those portions of general astronomy which are relevant to the main purpose. The chapters on time and on precession and nutation seem particularly clear and good. That on aberration follows the traditional lines of Gauss and Bessel, and criticism would be out of place here. Yet the exposition of Gauss, which seems to assume the apparent composition of the velocities of light and of the earth as a matter of course, appears to be imperfect in view of the difficulties in the physical theory. Is it not more logical to consider the apparent composition as an inductive result instead of the explanation of the astronomical phenomena?

The second part contains a discussion of undisturbed heliocentric motion. Dr. Bauschinger asserts (p. 170) that Lambert's equation is of little use in the case of ordinary elliptic orbits. This opinion may be disputed. It is true that the development in series is of little assistance owing to slow convergence, but in its original form the equation can be easily solved in all ordinary cases. The natural expression of the formulæ for motion in a hyperbola involves hyperbolic functions. The use of these is entirely avoided, presumably because tables of hyperbolic functions are not as a rule accessible to the computer.

The properties of the apparent or geocentric motion are discussed in the third part. Here will be found Bruns' elegant proof of the theorem of Lambert on the curvature of the apparent orbit. Incidentally it may be remarked that Lambert seems to have missed that measure of fame to which his unquestionable eminence as a mathematician entitles him.

The longest part is the fourth, in which the various methods of determining a preliminary orbit are described. An excellent feature is the compendious arrangement of the working formulæ. This part is followed by that on the adjustment of an orbit by the method of least squares. In both sections numerical examples are fully and clearly worked out.

The sixth part contains the theory of special perturbations. Three methods are given, according to which the perturbations can be calculated in the elements, or in polar or in rectangular coordinates. In the preliminary chapter, on mechanical integration, the usual German notation for interpolation formulæ is employed. It is difficult to see the advantage of this over the ordinary notation of finite differences. The last chapter of this section brings the reader to the determination of the definitive orbit.

Here the work might have ended, but Dr. Bauschinger has added a final part, in which he investigates the determination of the orbits of meteors, satellites, and double stars. These last chapters are necessarily brief, and it may be doubted whether, as regards unity of subject, their inclusion is justified. But that on satellites is certainly valuable, especially in view of recent discoveries.

The source of the numerous theorems which are met with in the work has generally been indicated, but this is not always the case. Thus the theorems on p. 184 are due to M. Radau (Bull. Astr., x. p. 11) and to Mr. Shin Hirayama (Monthly Notices, R.A.S., lxii., p. 620). Such references add greatly to the interest, but of course it is always difficult to be sure that the sources are strictly original. For instance, the proposition attributed (p. 131) to van der Kolk was, as has been recently pointed out, previously given by Whewell. There is an index at the end of the volume, but it is not so complete as it should have been. A full index of names is needed.

An outline of the method of Gibbs will be found in Dr. Bauschinger's work, but for fuller details the pamphlet of Dr. Frischauf may be consulted with advantage. The method is based on the use of a particular expression for the ratio of a triangle to the corresponding sector of an ellipse. The form is mathematically elegant and the degree of approximation is high, but it was thought to entail greater complexity in the computations, while, on the other hand, the method by itself gave little assistance when a still closer approximation proved necessary. This defect was remedied by Prof. Harzer. The modified method is described by Dr. Frischauf in a clear and interesting manner; the practical value of his account would have been enhanced by the addition of a fully worked example. The pamphlet also contains a number of supplementary notes to the author's "Grundriss der theoretischen Astronomie," a work of which a second edition appeared in 1903 after an interval of thirty-two years from its first publication. H. C. P.

INDUCTION AND CONDUCTION MOTORS.

Moteurs a Collecteur a Courants alternatifs. By Dr. F. Niethammer. Pp. 131. (Paris: *L'Éclairage Électrique*, 1906.)

THE title leads one to believe that the author is going to deal with at least all the principal types of modern alternate-current commutator motors, whereas the book is practically restricted to a consideration of the series induction and conduction motors. Shunt induction motors of the commutator type are occasionally touched upon, but all remarks concerning these must be considered as quite erroneous. Generally speaking, the number of mistakes is too great.

In chapter i. the historic part does not deal with the machines out of which those modern single-phase commutator motors have been directly evolved, which are afterwards considered more closely. The preliminary consideration of some of the types now in use is full of errors, and much prominence is given to the least important of these types. The indiscriminate use of the expression "repulsion" motor leads to the usual confusion.

In the second chapter, which is the most important in the whole book, we find the author trying to

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establish exact diagrams which will cover all types of motors. It may be possible to achieve this, but the task is not an easy one, and the solution offered by the author can certainly not be accepted. Take the two simple diagrams Figs. 32 and 33; the first illustrates the action of the motor shown in Fig. 30, the second (which is not referred to in the text) is probably intended to illustrate the action of the motor shown in Fig. 31. The E.M.F. (J_aW_a) in Fig. 32 is responsible for the current J_a flowing in the short-circuited rotor; it must therefore be the resultant of all those E.M.F.'s which are effective so far as the short-circuiting brushes are concerned. These E.M.F.'s are E_r, E_i, E_r', E_i', and E_s'. When the motor is standing, E_r and E_r' are nil, but they increase in direct proportion with the speed, with the result that J_aW_a must, according to the diagram, increase with the speed independently of the load! In other words, the rotor current J_a must increase with the speed, consequently also the stator current J_f. Seeing that the machine is one with a series characteristic, it is very obvious that the diagram in question cannot be correct. In a machine of the kind the tendency of the current is, of course, to diminish with the speed. The fact of the matter is that the phase of E_r' is shown incorrectly. If the direction of rotation is such that E_r is in phase with the flux K_a, then E_r' must be of opposite phase to the flux K_q, for these fluxes are not only at right angles to each other in space, but also nearly at right angles to each other in phase. The presence of this very serious mistake evidently prevented the author from grasping the full meaning of the various vectors of his diagram. (E_i) must be considered as the working E.M.F.; (E_r') is then the back E.M.F., ($E_s'+E_i'$) represents the self-induction of the rotor circuit, whilst E_r (and not E_r', as stated by the author on p. 32) must be looked upon as the compensating E.M.F. It is nearly opposed to ($E_s'+E_i'$), therefore tends to cancel the effect of the self-induction in the rotor and to bring J_a more and more into phase with E_i. Since E_r increases with the speed, it follows that with increasing speed the phase of J_a will approach that of E_i, and that the power factor will rapidly improve.

The writer also fails to agree with the author's Fig. 33. Owing to a mistake similar to that present in Fig. 32, we get the following curious and impossible result. It is obvious that E_r, which appears at the brushes (*aa*), must be responsible for the flux K_q; it is generally admitted that a magnetic field *lags* by about 90 degrees behind the E.M.F. responsible for it, yet in Fig. 33 K_q actually *leads* E_r by nearly that amount. The author also ignores the fact that for the arrangement of brushes shown in Fig. 31 we have *two* currents in the rotor, the one flowing from (*b*) to (*b*), the other from (*a*) to (*a*), the former being the working current, the latter producing K_q.

The value of the next fundamental diagram (Fig. 35) is greatly reduced because the author mistakes, in Fig. 34, the axis K_a for the axis K_q, thus making a comparison between Fig. 34 and Figs. 30 and 31 quite impossible. In Fig. 34 the motor-field axis K_q is the *vertical* axis, and *not* the *horizontal*, as has been assumed by the author, so that E_r is not nil as stated. No more is E_i nil, although it is now impressed on the rotor by conduction, and not by induction, as in Figs. 30 and 31.

The writer's space is limited, and he must therefore cut his remarks short. The fundamental diagrams of chapter ii. having been proved to be wrong, the value of the whole chapter is naturally greatly discounted. The chapter, however, contains a number of other mis-statements, some of which we will note in passing.

On p. 34 it is stated that the transformer flux in a short-circuited transformer is zero! On p. 42 that the motor shown in Fig. 53 is compensated in the same manner as the Winter-Eichberg machine, whereas compensation is due to the alteration brought about in the phase of the motor field by the introduction into the exciting circuit of the auxiliary E.M.F. derived from S_1. In diagram 43 the E.M.F. (E_r') is shown as being of opposite phase to the E_r' of Fig. 32, although both diagrams refer to the same motor. The remarks on commutation are difficult to follow, because of the attempt to deal with the various types of motors at one and the same time. It is recommended that flux K_a should be chosen low at starting for motors of the series induction type, whereas it is the flux K_q which at that time should be small. Contrary to the author's statement, the commutation difficulties with polyphase commutator motors are just about of the same order as those met with in the series induction motor. In dealing with the power factor (p. 59), the author makes a statement in the last paragraph which reveals a great confusion of ideas. This mistake probably arises out of the confusion of the axes of K_q and K_a already pointed out in connection with Fig. 34; in addition, the notation is now suddenly changed. It is, however, evident that for the case of the series conduction motor (Fig. 34) k_g stands for the field coaxial with the armature brushes and due to the armature ampere turns; k_g is perpendicular to K_q, and by neutralising k_g as shown in Fig. 37 or 39 the power factor is improved as stated. But in a "repulsion" motor such as Fig. 30, k_g does not exist; it is neutralised *ipso facto* because the energy is conveyed into the rotor by induction, and not by conduction as in Fig. 34. Furthermore, if k_g did exist, it would be coaxial with K_a. If k_g is a misprint for K_q, then by neutralising it the torque of the motor would be destroyed, for K_q is the *motor field.* As to speed regulation, and contrary to the author's opinion, any so-called repulsion motor can be satisfactorily controlled by suitably influencing the rotor circuits.

Chapter iii. only deals with motors full descriptions of which have appeared from time to time in the technical Press. As to the notes on the predetermination of alternate-current commutator motors, these are very superficial, and mainly apply to the series conduction machine.

On the whole, the book is more likely to bewilder the reader than teach him anything; it ought to be very thoroughly revised and corrected before it can

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be recommended. The author will find it easier and more profitable to treat each type of motor separately, and then to point out the differences between the various types, than to try and establish diagrams and formulæ which will meet all cases.

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SUBAQUEOUS TUNNELLING.

Tunnel Shields, and the Use of Compressed Air in Subaqueous Works. By W. C. Copperthwaite. Pp. xv+390. (London: Archibald Constable and Co., Ltd., 1906.) Price 31s. 6d. net.

THIS fine quarto volume furnishes a very valuable and comprehensive history of a system of tunnelling, especially under rivers and in water-bearing strata, which was inaugurated by Sir Marc Isambard Brunel, as regards the employment of a shield, in the celebrated Thames Tunnel between Rotherhithe and Wapping, commenced in 1825, but, owing to the inrush of the river into the works on two occasions through breaks in the stratum of clay, and financial difficulties, only completed in 1843.

The second important step in the development of the system in a practical form was, curiously enough, taken in constructing a second tunnel under the Thames rather higher up the river, crossing just above the Tower, which was commenced in February, 1869, and completed in November the same year. This Tower Subway, originally proposed by Mr. Peter Barlow, but eventually executed by the late Mr. Greathead, whose name will always be prominently associated with the system of tunnelling under consideration, was carried forward through the London Clay under the shelter of a shield, similar in principle to, though much smaller than, the Thames Tunnel shield. The shield in this instance consisted of a short wrought-iron cylinder laid horizontally, $4\frac{3}{4}$ feet long and slightly more than 7 feet internal diameter, stiffened at its front cutting-edge, and provided inside with a vertical plate diaphragm having a central opening, which could be readily closed, through which the men passed for excavating the ground in front preparatory to pushing forward the shield by a series of screws. The novelty consisted in the lining of the tunnel being formed of a series of cast-iron rings, composed of segments bolted together, which were erected under the shelter of the rear part of the cylindrical portion of the shield as it was pushed forward; and as the shield overlapped the lining of the tunnel, and left a slight annular space between the lining and the clay stratum, lime grout was injected through holes provided in the casting, so as to fill up the vacancy left by the shield in its advance. This subway traverses the London Clay throughout, at a minimum depth of 22 feet below the river-bed, no water having been encountered; and it indicates the general method of constructing tunnels by this system. The shield serves to protect the completed end of the tunnel from the fall of earth at the working face, and acts like timbering in supporting the superincumbent mass and preventing settlement above during construction, which is further insured over the completed tunnel by filling the cavities left by the advancing shield with grout.

The system, however, as successfully carried out, in the absence of water, in the Tower Subway, was not adapted for passing through water-bearing strata; and a third step, consisting in the introduction of compressed air, was essential to enable this system to cope effectually with the conditions liable to be encountered in tunnelling under rivers, or at a considerable depth below the surface, in loose ground. The completion of this system of tunnelling, by the combined use of a shield, a cast-iron lining put together under shelter of the shield, and compressed air to exclude the water from the works in traversing water-bearing strata, has enabled abandoned tunnels to be completed, and tunnels to be successfully carried out under such unfavourable conditions as would have been considered impracticable by the methods previously in use. This combination of shield, cast-iron lining, and compressed air, for carrying a tunnel through water-bearing strata, was resorted to by Mr. Greathead for the first time in 1887, in constructing the City and South London Railway, the first of the metropolitan tube railways, where it passes through the loose, water-logged gravel of the Thames basin, overlying the London Clay; and in 1889 it was adopted for continuing the Hudson Tunnel in the silt underlying the Hudson River separating New York from the mainland, when different systems of carrying forward an iron lining by the aid of compressed air, under the shelter of which a brick tunnel was constructed, proved increasingly difficult as the work advanced.

The shield for the continuation of the two single-line Hudson tunnels was $10\frac{1}{2}$ feet long and 20 feet outside diameter; whilst the cast-iron lining has an external diameter of $19\frac{1}{2}$ feet and 18 feet internal diameter, formed of rings $1\frac{1}{2}$ feet long, made up of eleven segments and a key, put in place by a revolving hydraulic erector. This work was stopped for want of funds in 1891, but was resumed in 1903 and completed last year. Where the silt traversed was very soft, the shield was kept closed and pushed forward by sixteen hydraulic rams; and to avoid unequal settlement of the tube under the weight of a train, it has been supported at intervals on iron piles driven down to a hard stratum underlying the silt. Compressed air had been used successfully for many years in constructing foundations and piers of bridges under water, or in water-bearing strata, before it was applied to subaqueous tunnelling; but whereas in bottomless, vertical caissons, the compressed air forces out the water uniformly all over the bottom, the pressure of the air at the open end of a horizontal tube meets with less opposition from the water at the top than at the bottom, where the head of water is greater, in proportion to the diameter of the tube. Accordingly, in large tubes there is a liability in traversing loose soil for the air to escape through the stratum at the top, and for the water to rush in simultaneously at the bottom. To provide for the safety of the men in such a contingency, in addition to two or three platforms at the back of the diaphragm of the shield, with openings at each stage which can

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